Homework 9

Page 191 - 2. Determine whether each of these functions is $O(x^2)$.
   a) $f(x) = 17x + 11$
   b) $f(x) = x^2 + 1000$
   c) $f(x) = x \log x$
   d) $f(x) = x^{4/2}$
   e) $f(x) = 2^x$
   f) $f(x) = \text{floor}(x) \times \text{ceiling}(x)$

Page 191 - 8. Find the least integer $n$ such that $f(x)$ is $O(x^n)$ for each of these functions.
   a) $f(x) = 2x^2 + x^3 \log x$
   b) $f(x) = 3x^3 + (\log x)^4$
   c) $f(x) = (x^4 + x^2 + 1)/(x^4 + 1)$
   d) $f(x) = (x^3 + 5 \log x)/(x^4 + 1)$

Page 199 - 4. Determine the number of multiplications used to find $x^{2^k}$ starting with $x$ and successively squaring (to find $x^2, x^4, x^8$, and so on). Is this a more efficient way to find $x^{2^k}$ than by multiplying $x$ by itself the appropriate number of times?

Page 208 - 6. Show that if $a, b, c,$ and $d$ are integers such that $a \mid c$ and $b \mid d$, then $ab \mid cd$.

Page 209 - 10. What are the quotient and remainder when
   a) 44 is divided by 8?
   b) 777 is divided by 21?
   c) -123 is divided by 19?
   d) -1 is divided by 23?
   e) -2002 is divided by 87?
   f) 0 is divided by 17?
   g) 1,234,567 is divided by 1001?
   h) -100 is divided by 101?

Page 217 - 4. Find the prime factorization of each of these integers.
   a) 39
   b) 81
   c) 101
   d) 143
   e) 289
   f) 899

Page 217 - 12. Determine whether the integers in each of these sets are pairwise relatively prime.
Page 218 - 20. What are the greatest common divisors of these pairs of integers?

   a) \(2 \cdot 3^2 \cdot 5^2\), \(2^5 \cdot 3^3 \cdot 5^2\)
   b) \(2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13\), \(2^{11} \cdot 3^9 \cdot 11 \cdot 17^{14}\)
   c) 17, 17
   d) \(2^2 \cdot 7\), \(5^3 \cdot 13\)
   e) 0, 5
   f) \(2 \cdot 3 \cdot 5 \cdot 7\), \(2 \cdot 3 \cdot 5 \cdot 7\)

Page 218 - 28. Find the smallest positive integer with exactly \(n\) different factors when \(n\) is

   a) 3.
   e) 6.
   b) 4.
   f) 10.
   c) 5.